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How to Construct a 3D Mathematical/Computer Model of the Left Ventricle

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Background: How can mathematics help us to understand the mechanism of the cardiac motion? The best known approach is to take a mathematical model of the fibered structure and insert it into a more-or-less complex model of a cardiac architecture.
Objectives: We provide a new mathematical tool by introducing the notions strains, which are two-by-two and three-by-three matrices.
Materials and Methods: Using motion and deformation echocardiographic data, force vectors of myocardial samples were estimated by MATLAB software, interfaced in the echocardiograph system. Dynamic orientation contraction (through the cardiac cycle) of every individual myocardial fiber could be created by adding together the sequential steps of the multiple fragmented sectors of that fiber.
Results: Myocardial fibers initiate from the posterior basal region of the heart, continue through the left ventricular free wall, reach the septum, loop around the apex, ascend, and end at the superior-anterior edge of the left ventricle.
Conclusions: These studies will enable physicians to diagnose and follow up many cardiac diseases when this software is interfaced within echocardiographic machines.

Keywords: Echocardiography system; Mathematical Modeling; MATLAB Software; Left Ventricular Myocardium; Deformation Map

1. Background

In a great many problems the microscopic structure of matter can be disregarded and a biological body replaced by a continuous mathematical model whose geometrical points are identified with material points of the body. The learning of such models is in the province of the mechanics of continuous media, which covers a vast range of problems in elasticity, hemodynamics, aerodynamics, plasticity, and electrodynamics. When the relative position of points in a continuous biological body is altered, we say that the body is strained. The change in the relative position of a point is a deformation, and the study of deformations is the region of the analysis of strain. In this paper, the left ventricular myocardium is taken as a biological elastic body and modeled by mathematical/imaging techniques (1-8).

2. Objectives

We provide a new mathematical tool by introducing the notions strains, which are two-by-two and three-by-three matrices. Entries of the proposed matrices are provided by myocardial velocity and strain components. The area of the cross-section of a myocardial sample is the determinant of our two-by-two matrix, and the volume of the same myocardial sample is the determinant of our three-by-three matrix. Through these data as well as some new mathematical techniques and formulas, it would be possible test the helix motion of the left ventricle (LV). The results we have obtained so far confirm this postulate (William Harvey's hypothesis: the helical orientation of cardiac fibers and interest in fiber architecture).

3. Materials and Methods

The LV as an elastic muscular object is studied in the theory of mathematical elasticity. Material points or muscle volume samples of the LV are replaced with the geometrical points of an elastic object which acts as a mathematical modeling of the LV. Currently, an echocardiogram presents the left ventricle (LV) based on images obtained from ultrasound methods. Utilizing mathematical equations, specific echocardiographic data may provide more detailed, valuable and practical information for physicians. In our study using appropriate mathematically based software, we have attempted to create a novel software capable of demonstrate LV model in normal hearts (Figure 1).

Echocardiographic data of a myocardial muscle sample include velocity, displacement, strain rate, and strain, which explain the motion and deformation of the muscle volume sample. Echocardiography was performed on healthy volunteers. Data evaluated included the velocity (radial, longitudinal, rotational, and vector point),

Implication for health policy/practice/research/medical education:
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displacement (longitudinal and rotational), strain rate (longitudinal and circumferential), and strain (radial, longitudinal, and circumferential) of all 16 LV myocardial segments. Using these data, the force vectors of the myocardial samples were estimated by MATLAB software interfaced in the echocardiographic system. (MATLAB is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation. MATLAB can solve technical computing problems faster than can traditional programming languages such as C, C++, and Fortran. MATLAB can be employed in a wide range of applications, including signal and image processing, communications, control design, test and measurement, financial modeling and analysis, and computational biology). Having attached strain components (radial strain, longitudinal strain and circumferential strain) to each myocardial muscle sample, we defined a flat and smooth map by the following function (Figure 2).

Fibers of this map, $f^{-1}\{(*,*)\}$ = {those myocardial muscle samples that go to (*,*) under f} are those sets followed by a coded algorithm in the MATLAB software.

4. Results

Mathematical/physical assessments of the motion and deformation of the basal anterior, mid anterior, and apical anterior in the MATLAB software:

Let $\varepsilon_{rr,PbA}$, $\varepsilon_{ll,PbA}$ and $\varepsilon_{cc,PbA}$ be strain components of the basal anterior P_{bA}.

we set:

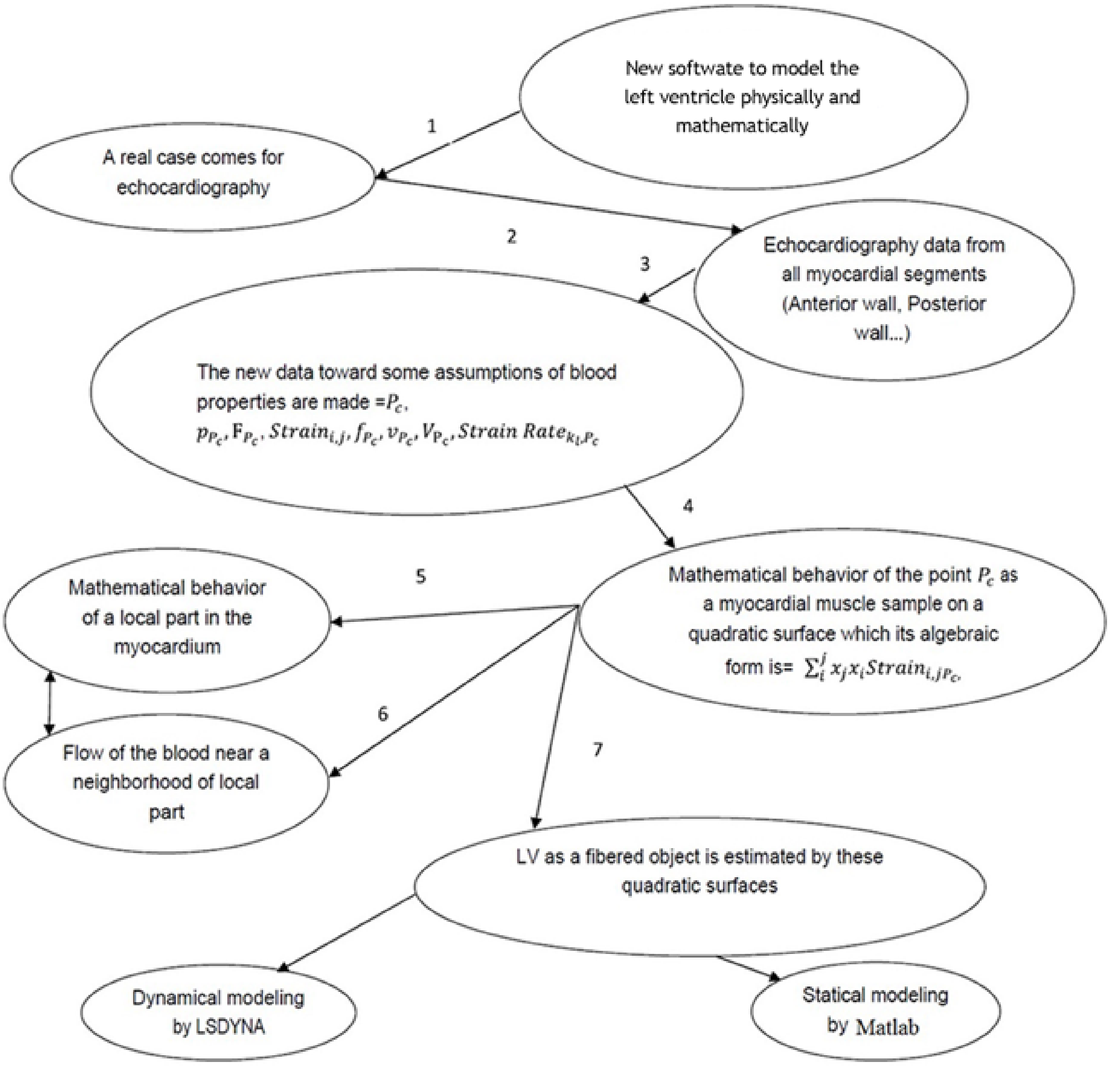

Figure 1. This figure Shows a General Flowchart of the Whole Mathematical Left Ventricular Modeling.

$$\gamma_{P_{bA}} = \{\text{each myocardial sample } X \text{ that } \varepsilon_{rr,X} \times \varepsilon_{ll,X} = \varepsilon_{rr,P_{bA}} \times \varepsilon_{ll,P_{bA}} \text{ and } \varepsilon_{rr,X} \times \varepsilon_{ll,X} \times \varepsilon_{cc,X} = \varepsilon_{rr,P_{bA}} \times \varepsilon_{ll,P_{bA}} \times \varepsilon_{cc,P_{bA}}\}$$

In fact γ_{PbA} is that which is passed from the basal anterior with algebraic equations:

$$Q_{P_{bA}}\colon D_{P_{bA}} = \left(\sum_{k,l} \varepsilon_{rr}'_{P_k P_l}\, dt\right) . y_1^2 + \left(\sum_{k,l} \varepsilon_{ll}'_{P_k P_l}\, dt\right) . y_2^2 + \left(\sum_{k,l} \varepsilon_{cc}'_{P_k P_l}\, dt\right)$$

Where P_k and P_l are points belonging to γ_{PbA}.

The other segments are studied with the same argument.Gluing together these myofiber bands can confer a new physical modeling of the LV. The coded algorithm due to following fibers of Map f is rendered in MATLAB. It shows that myocardial fibers initiate from the posterior basal region of the heart, continue through the LV free wall, reach the septum, loop around the apex, ascend, and end at the superior-anterior edge of the LV (Figure 3).

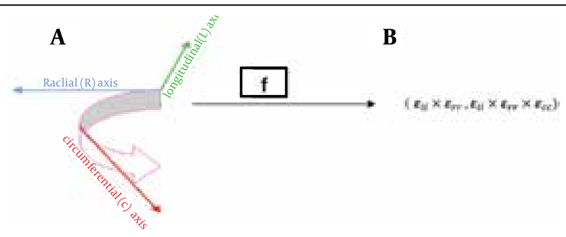

Figure 2. Strain Components Attached to a Myocardial Sample (A). How to Map a Myocardial Sample to Two-Dimensional Space (B).

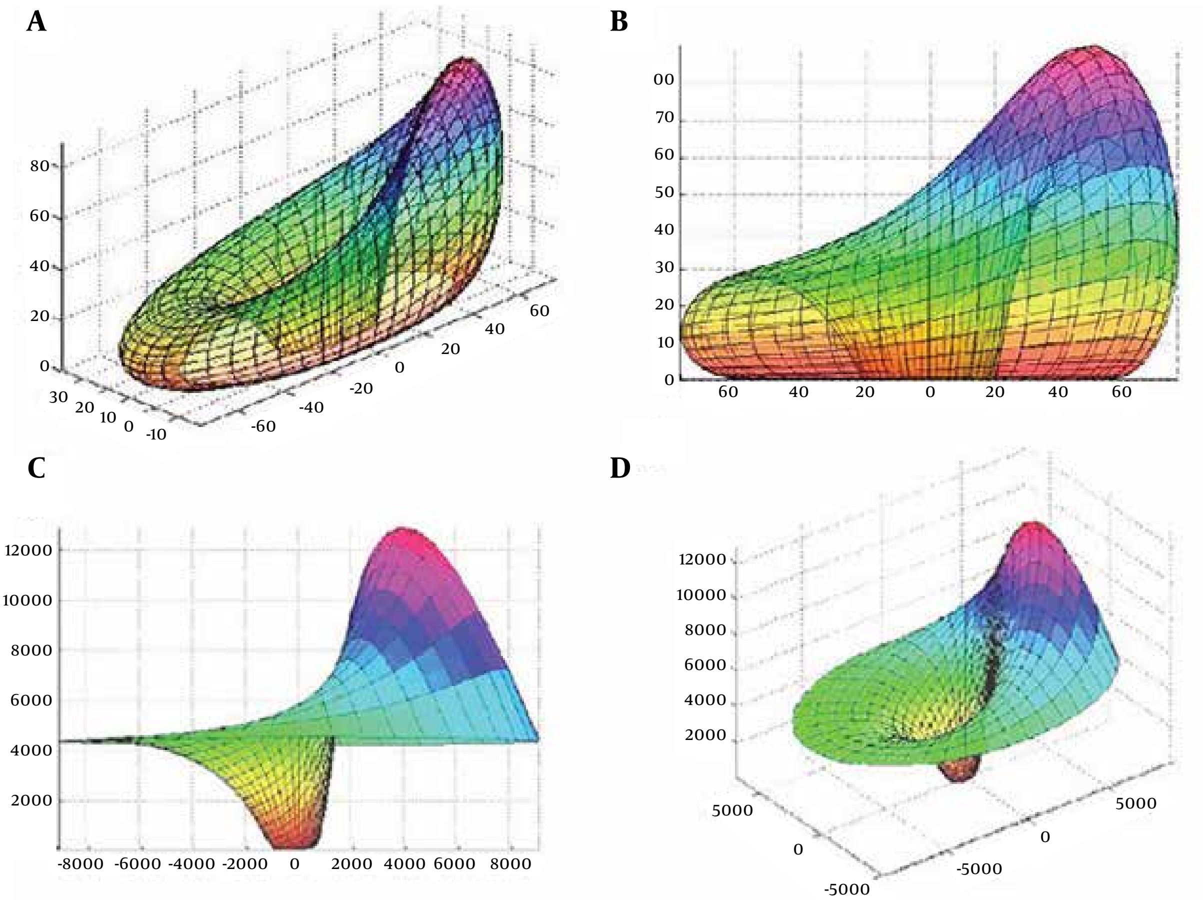

Figure 3. This figure Shows How We Mathematically Reconstruct the Left Ventricle and the Curves on It Step by Step in the MATLAB Software (A to D). Horizontal axis: the width of coordinates; Vertical axis: the length of coordinates.

5. Discussion

Velocity and strain are normally measured by echocardiography. This method has developed a method for measuring the power of the myocardium. Knowledge of the power of the myocardium has significant application in the diagnosis of cardiac diseases. As a follow-up to the research, we advanced a method for calculating the heat transfer from the myocardium to the blood. Knowledge of the heat transfer is important in understanding the chemical content of blood and in particular the release of natriuretic peptide in the heart.

Scientific advances during the last century have been concerned principally with the problems of the existence of solutions and the integration of several broad categories of the boundary-value problems. The direction of blood flow inside the LV starting with the mitral valve and ending up to the aortic valve was determined by sophisticated mathematical/physical models: modeling of LV, then using fluid mechanical (Navier-Stocks) equations through spectral theory (8, 9).

Previous investigations have strongly used searching points by image-processing machinery method, but our study demonstrates regional and global of the myocardial function by introducing a mathematical fibered model of it, which is rendered by a piece of software. This method is also very robust, reproducible, and user friendly. Its clinical values remain to be established, however (10-15). As another application of the method described in this paper, we can model the motion of blood in the ventricles. This initiation is for normal and diseased hearts. The model can be used for instructional purposes and diagnosis of heart ailments (16). The standard way to model the motion of blood inside the LV would be to treat the LV as an elastic membrane obeying Newton's laws of motion with forces calculated in part from the elasticity of the membrane and in part by evaluating the fluid stress tensor on the surface of the membrane. Then the fluid equations would have to be supplemented by the constraint that the velocity of the fluid on either side of the membrane must agree with the instantaneously known velocity of the elastic membrane itself. There is a difficulty with this standard approach to the problem. Challenge is the practice of evaluating the fluid stress tensor on either side of the boundary. This seems difficult (or at least messy) to do numerically, unless the computational grid is aligned with the boundary. On the other hand, in a moving boundary problem, it is both expensive and complicated to re-compute the grid at every time step in order to achieve alignment. This means that the sum of the elastic force and the fluid force on any part of the boundary has to be zero. Once we know this, it becomes unnecessary to evaluate the fluid stress tensor at the boundary at all! We can find the force of any part of the boundary on the fluid by evaluating the elastic force on that part of the boundary. Note the use of Newton's third law: the force of boundary on fluid is minus the force of fluid on boundary. All we need is a method for transferring the elastic force from the boundary to the fluid. On a Cartesian grid, this may be done by spreading each element of the boundary force out over nearby grid points. The fundamental quantity that describes the motion of the fluid is represented by the vorticity defined as the tendency of fluid elements to spin; more precisely, vorticity can be related to rotation of fluid elements and the formation of circulatory areas. Quantitative parameters of the intraventricular vortex were also extracted on the basis of the vorticity (curl $v \neq 0$, rotational blood flow is happened). This approach can be used for instructional purposes and diagnosis of heart ailments (17, 18).

In this article, we were able to define how to reconstruct the LV myocardium mathematically, by MATLAB software, in normal tissues. These data potentially allow the implementation of an image-based approach for patient-specific modeling of LV.

Author's Contributions

Mathematical modeling was done by Dr. Saeed Ranjbar, medical and clinical interpretations were done by Dr. Mersedeh Karvandi, Professor Seyed Ahmad Hassantash, and Dr. Mahnoosh Foroughi.

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